



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 22

Key Rate Duration

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KEY RATE DURATION



PRICE CHANGE WITH DURATION MEASURE

- Using the concept of duration, the percentage price change of an instrument is given by:

$$\frac{dP}{P} = -D \frac{dy}{1+y} \text{ where } D = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y)^t}}{P_0}$$

- Two important features are seen from the above formula:
- Duration at any instant is calculated on the basis of the YTM of the bond at that instant.
- Thus, it uses one single rate only and does not consider the individual spot rates corresponding to all the different cashflows from the bond i.e. it does not consider the term structure of interest rates or equivalently assumes a flat term structure.

- dy is also assumed the same across all the cashflows i.e. the formula assumes a PARALLEL shift in the entire yield curve by the same amount at each maturity.
- Thus, the duration is an adequate measure of bond price risk only for parallel shifts in the yield curve.
- Please note that convexity is a correction for the curvature of the yield price curve and not for the term structure of interest rates.

- The impact of nonparallel shifts can be measured using a concept known as key rate duration.



KEY RATE DURATION

- A key rate duration, also known as a partial duration, is defined as the sensitivity of the value of a bond or portfolio to *changes in the spot rate for a specific maturity, holding other spot rates constant.*

- **When keeping other maturities constant, the key rate duration is used to measure the sensitivity in a debt security's price to a 1% change in yield for a specific maturity.**
- **The effect on the overall portfolio is the sum of these individual effects.**

- **A bond or portfolio will have a key rate duration for each relevant maturity range on the spot rate curve.**
- **We can use the key rate duration for each maturity to compute the effect on the portfolio of the interest rate change at that maturity.**

- When a yield curve has a parallel shift, we can use Macaulay duration, but key rate duration must be used when the yield curve moves in a non-parallel manner, to estimate portfolio value changes.

COMPUTATION OF KRD

- The process of computing key rate duration is similar to the process of computing effective duration described earlier, except that instead of shifting the entire benchmark yield curve, **only one specific par rate (key rate) is shifted before the price impact is measured.**



$$\text{Key Rate Duration} = \frac{P(S_{0t} - \Delta S) - P(S_{0t} + \Delta S)}{2 \times \Delta S \times P(S_{0t})}$$

or

$$\frac{P(S_{0t} - 0.01) - P(S_{0t} + 0.01)}{2 \times 0.01 \times P(S_{0t})}$$

CALCULATING KEY RATE DURATION: EXAMPLE

- Assume that a bond is originally priced at \$1,000, and with a 1% increase in key yield would be priced at \$970, and with a 1% decrease in yield would be priced at \$1,040. The key rate duration for this bond would be:
- $KRD = (\$1,040 - \$970) / (2 \times 1\% \times \$1,000) = \$70 / \$20 = 3.5$
- where: KRD=Key rate duration .

UTILITY OF KEY RATES

- For example, assume bond X has a one-year key rate duration of 0.5 and a five-year key rate duration of 0.9.
- Bond Y has a key rate durations of 1.2 and 0.3 for these maturity points, respectively.
- What inference do these figures provide?

- It could be said that bond X is half as sensitive as bond Y on the short-term end of the curve, while bond Y is one-third as sensitive to interest rate changes on the intermediate part of the curve.

IMPORTANCE OF KEY RATE DURATION

- Key rate durations or partial durations capture the interest rate sensitivity of a bond to changes in yields (par rates) of specific benchmark maturities.
- Key rate duration is used to identify the interest rate risk from changes in the shape of the yield curve (shaping risk).

PROPERTIES OF KRD: PAR BONDS

- For an option-free ZCB, the bond's maturity-matched rate is the only rate that affects the bond's value.
- Its maturity key rate duration is the same as its effective duration, and all other rate durations are zero.

PROPERTIES OF KRD: NON-PAR BONDS

- For an option-free bond not trading at par, the maturity-matched rate is still the most important rate because the largest cash flow from the bond is the maturity cash flow..
- A bond with a low coupon rate may have negative key rate durations for horizons other than the bond's maturity.

PROPERTIES OF KRD: DECREASE IN YTM

- Consider bonds X and Y each of maturity 2 years and face value 1,000. X is a ZCB and Y is a 20% coupon bond with both being traded at 20% YTM.
- The price at $t=0$ of X is $\frac{1,000}{1.2^2} = 694.44$ while that at $t=1$ is $\frac{1,000}{1.2} = 833.33$. If the YTM changes to 15% at $t=1$, the bond price will change to $\frac{1,000}{1.15} = 869.56$ i.e. an increase of 36.23.

- For the bond Y, the price at $t=0$ & $t=1$ are both 1,000 since it is a par bond. If the YTM changes to 15% at $t=1$, its price changes to $\frac{1,200}{1.15} = 1043.48$ i.e. an increase of 43.48.
- Thus, other things being equal (YTM & FV), larger the coupon, larger is the increase in price for a given fall in YTM.

PROPERTIES OF KRD: INCREASE IN YTM

- Consider bonds X and Y each of maturity 2 years and face value 1,000. X is a ZCB and Y is a 20% coupon bond with both being traded at 20% YTM.
- The price at $t=0$ of X is $\frac{1,000}{1.2^2} = 694.44$ while that at $t=1$ is $\frac{1,000}{1.2} = 833.33$. If the YTM changes to 25% at $t=1$, the bond price will change to $\frac{1,000}{1.25} = 800.00$ i.e. a decrease of 33.33.

- For the bond Y, the price at $t=0$ & $t=1$ are both 1,000 since it is a par bond. If the YTM changes to 25% at $t=1$, its price changes to $\frac{1,200}{1.25} = 960.00$ i.e. a decrease of 40.00.
- Thus, other things being equal (YTM & FV), larger the coupon, larger is the decrease in price for a given rise in YTM.

- Thus, a callable bond with a low coupon rate is unlikely to be called since its price would not increase enough due to a decrease in interest rate changes.
- Hence, the bond's maturity-matched rate is the most critical rate (i.e., the highest key rate duration corresponds to the bond's maturity).
- All else equal, higher coupon bonds are more likely to be called, and therefore in their case the time-to-exercise rate will tend to dominate the time-to-maturity rate.

ONE SIDED DURATION

- The computation of effective duration discussed earlier relied on computing the value of the bond for equal parallel shifts of the yield curve up and down (by the same amount).
- This metric captures interest rate risk reasonably well for small changes in the yield curve and for option-free bonds.



- For a callable bond, when the call option is at or near the money, the change in price for a decrease in yield will be less than the change in price for an equal amount of increase in yield.
- Why, because the value of a callable bond is capped by its call price.
- Thus, the bond's value will not increase beyond the call price regardless of how low interest rates fall.
- Similarly, the value of a puttable bond is more sensitive to downward movements in yield curve versus upward movements.



- **For bonds with embedded options, one-sided durations—durations that apply only when interest rates rise (or, alternatively, only when rates fall)—are better at capturing interest rate sensitivity than simple effective duration.**
- When the underlying option is at the-money (or near-the-money), callable bonds will have lower one-sided down duration than one-sided up-duration.
- This is because when interest rates fall, call option will come into play to truncate the value of the bond at the call exercise.



- **Thus, the price rise of a callable when rates fall is smaller than the price fall for an equal increase in rates.**
- **Conversely, a near-the money puttable bond will have larger one-sided down-duration than one-sided up-duration.**